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SPATIAL MAJORITY VOTING EQUILIBRIA AND THE PROVISION OF PUBLIC GOODS

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Spatial Majority Voting Equilibria and

the Provision of Public Goods

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Running Head: Majority Voting Equilibria

Abstract

This paper develops general equilibrium conditions for urban areas where a pure public good is provided at a level decided upon by majority voting. Models with a property tax and a head tax and external land ownership are analysed, and equilibrium solutions are compared using a specific form for the utility function. The optimal city with a head tax is characterized and the solution is compared to the majority voting equilibrium, again using a specific utility function. Models where aggregate land rent is divided equally among the urban residents are also developed.

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The purpose of this paper is to characterize spatial equilibrium in an urban area with a pure public good whose output is chosen by majority voting. Models with a property tax and a head tax and external land ownership are analysed, and majority voting equilibria are compared by computing solutions using a Cobb-Douglas utility function. The optimal city with the head tax is analysed and the solution is compared to the majority voting equilibrium with the head tax using a CES utility function. Models where aggregate land rent is divided equally among the urban residents are considered in the last part of the paper.

The available papers dealing with urban spatial equilibrium with a pure public good are unsatisfactory. Wile [3] presents a model with an extremely restrictive property tax formulation which drastically oversimplifies the consumer choice problem. Barr's analysis [1] also suffers from inadequate treatment of the decision process by which the level of the public good is set. This paper integrates, for the first time, models of majority voting and urban spatial equilibrium. In addition, the general equilibrium models in the paper represent an improvement over the partial equilibrium majority voting models analysed by authors such as Barr and Davis [2].

I.

The models in this paper incorporate standard features of urban spatial analysis. All consumers commute to the CBD, which is the only employment center

in the city, and the CBD output is exported at a fixed price. The CBD is a dimensionless point, and labor is the only input to CBD production. The wage, y, is independent of the level of employment by the assumptions of constant returns to scale in the CBD production process and perfect competition in the export market. The public good, z, is imported at a fixed unit price c, and the commuting cost function is linear in distance x from the CBD: $t(x) \equiv tx$. More general assumptions which would not restrict $t(x)$, permit y to depend on the level of CBD employment, and allow the unit price of z to depend on the level of z would greatly complicate or prevent model solutions using specific forms for the utility function.

A strong assumption is that people consume only land, ℓ, and the public good, which will imply that per capita land consumption is constant throughout the urban area, a radical departure from the normal conclusion that land consumption increases with distance from the CBD. Analysis of majority voting equilibria in a three-good model where per capita land consumption could vary over space would be prohibitively difficult. Until Section IV of the paper, it is assumed that the urban land rent is paid to absentee landlords.

Since z is a pure public good, its consumption does not depend on x. Because the consumer utility level must be invariant over x in equilibrium, the consumption of land cannot vary with distance in equilibrium. Thus, the urban equilibrium is characterized by a consumption bundle (ℓ,z) and a utility level $u = u(\ell,z)$. In addition, the equilibrium urban population n must be housed within the urban periphery $\bar{x}$, and land rent at the periphery must equal agricultural rent. Either u or n must be exogenous; the city is either open to migration so that n adjusts until the utility level equals the prevailing external level, or it is closed to migration with n fixed and u endogenously determined.

Since z is purely public, consumption of z is identically equal to output. Congestion would imply that consumption of z depends both on output and the size of the population consuming the output.

Two methods of financing the public good are considered: a property tax and a head tax. Consumers are assumed to know c, the unit price of the public good. The following definition formalizes the equilibrium concept used in the analysis.

> <u>Definition</u>: A majority voting equilibrium is a vector $e^* = (\ell^*, z^*, u^*, n^*)$ such that there does not exist a $z \neq z^*$ and a set of consumers N larger than $n^*/2$ such that for all $i \in N$, (ℓ_i, z) is affordable consumption bundle for consumer i and $u(\ell_i, z) > u^*$.

Any vector $\bar{e}$ not satisfying the definition will not be a majority voting equilibrium because there will exist a $z' \neq \bar{z}$ such that z' would attract the votes of a majority of the urban residents in a contest with $\bar{z}$.

We now analyze equilibrium with the property tax. The property tax rate θ which balances the city's budget is $\theta = cz/V$, where V is the total value of land in the urban area. The budget constraint of a consumer living at x is $r(x)\ell(1+\theta) = y - tx$, or

$$\ell = \frac{y - tx}{r(x)(1 + cz/V)}, \tag{1}$$

where r(x) is land rent at x. The consumer at x maximizes utility subject (1). The maximand is

$$u\left(\frac{y - tx}{r(x)(1 + cz/V)}, z\right) \tag{2}$$

Since consumers are perfectly competitive, they assume their consumption decision has no effect on V, aggregate land value. Land value V is endogenous in the model, but each consumer takes it as given in making his consumption decision.

The first-order condition is

$$\frac{u_2(\ell,z)}{u_1(\ell,z)} = \frac{(y-tx)c/V}{r(x)(1 + cz/V)^2} = \frac{c\ell}{V+cz}\,, \tag{3}$$

substituting for r(x) from (1).[2] Equation (3) does not involve x, which implies that all consumers are satisfied with the same bundle (ℓ^*, z^*) in equilibrium. Letting the equilibrium value of V equal V*, the equilibrium land rent function is $r(x) = (y-tx)/\ell^*(1 + cz^*/V^*)$, and thus the budget constraint in equilibrium is $\ell = \ell^*(1 + cz^*/V^*)/(1 + cz/V^*)$. The absolute value of the slope of the constraint, $|d\ell/dz|$, is $\ell^*(1 + cz^*/V^*)(c/V^*)/(1 + cz/V^*)^2$, which equals $c\ell^*/(V^* + cz^*)$ at (ℓ^*, z^*). Since (3) holds at (ℓ^*, z^*, V^*), the equilibrium budget constraint is tangent to the equilibrium indifference curve at (ℓ^*, z^*) for all x. This establishes that the majority voting equilibrium is one of voter unanimity; no individual i prefers an affordable bundle (ℓ_i, z) with $z \neq z^*$ to the equilibrium bundle (ℓ^*, z^*).

In equilibrium, the land rent function based on V must generate aggregate land value equal to V. That is, V must satisfy

$$\int_0^{\bar{x}} 2\pi x\ r(x)dx = V, \tag{4}$$

or, substituting for r(x) from (1),

$$\int_0^{\bar{x}} 2\pi x\ \frac{(y - tx)}{\ell(1 + cz/V)}\ dx = V. \tag{5}$$

Rearranging,

$$\int_0^{\bar{x}} \frac{2\pi x}{\ell} (y - tx)\ dx = V + cz, \tag{6}$$

which states that the integral of land area times disposable income per acre over the city should equal the total value of land plus the cost of the public good .

The complete set of equilibrium conditions for the city with a property tax is

$$\frac{y - t\bar{x}}{\ell(1 + cz/V)} = r_A \tag{7}$$

$$n\ell = \pi\bar{x}^2 \tag{8}$$

$$\int_0^{\bar{x}} \frac{2\pi x}{\ell} (y - tx)\ dx = V + cz \tag{9}$$

$$\frac{u_2(\ell,z)}{u_1(\ell,z)} = \frac{c\ell}{V+cz} \tag{10}$$

$$u = u(\ell,z) \tag{11}$$

Equation (7) states that land rent equals the agricultural rent r_A at the urban periphery, while (8) states that the city houses its population, and (11) gives the utility level. The unknowns are ℓ, z, u, $\bar{x}$, n, and V, and since there are only five equations, either u or n must be exogenous.

The striking feature of the property tax equilibrium is voter unanimity over the equilibrium consumption bundle. In the model with the head

tax, which is developed next, there is disagreement over the optimal consumption bundle in equilibrium; the majority voting process is central to the outcome. With the head tax, each individual pays an amount cz/n. The budget constraint of a consumer living at x is $r(x)\ell + cz/n = y - tx$ or

$$\ell = \frac{y - tx - cz/n}{r(x)} . \tag{12}$$

Substituting (12) in the utility function and computing the first-order condition[3] yields

$$\frac{u_2(\ell,z)}{u_1(\ell,z)} = \frac{c}{n\ r(x)}.$$

$$= \frac{c\ell}{n(y - tx) - cz} , \tag{13}$$

substituting for $r(x)$ from (12). In equilibrium, ℓ and z must be the same at all x. Thus the LHS of (13) is constant over x while the RHS is increasing in x, and (13) holds at only one value of x. This is illustrated in Figure 1, where the consumption bundle is (ℓ', z'), which implies that the land rent function is $r(x) = (y - tx - cz'/n)/\ell'$. In the Figure, the line C is the budget line of the consumer living at x', the x for which (13) holds when $\ell = \ell'$ and $z = z'$. Line D is the budget line of a consumer living at $x > x'$, where $r(x) < r(x')$, and lines A and B are budget lines for consumers living inside x'. Suppose x' is greater than the median distance of consumers from the CBD, the distance $\tilde{x}$ such that $n/2$ consumers live inside $\tilde{x}$. Then the number of consumers who have budget lines like A and B in Figure 1 will exceed $n/2$. In this situation, the point f cannot be a majority voting equilibrium. To see

this, note that each individual having a budget line with slope less in absolute value than that of line A will prefer z" and the associated ℓ value on his budget line (a bundle such as b) to the bundle (ℓ', z'). Since x' is greater than the median distance $\tilde{x}$, we can always find a z" and a corresponding budget line A such that the number of consumers with budget lines less steep than A exceeds n/2. This means that in a contest between z' and z", z" will receive the votes of a majority of the population, establishing that point f could not have been a majority voting equilibrium. The same argument can be made if x' is less than the median distance $\tilde{x}$. We have shown that a majority voting equilibrium must have the property that (13) holds with $x = \tilde{x}$. In equilibrium, the consumer at $\tilde{x}$ is satisfied with his consumption bundle, while, at the equilibrium land rents, consumers inside $\tilde{x}$ want less ℓ and more z and consumers outside $\tilde{x}$ desire more ℓ and less z.

Since land consumption is constant, $\tilde{x}$ can be easily expresed in terms of $\bar{x}$, the distance to the urban periphery. By the definition of $\tilde{x}$, the number of people living inside $\tilde{x}$, or $\pi\tilde{x}^2/\ell$, equals n/2, or $\pi\bar{x}^2/2\ell$. Thus $2\tilde{x}^2 = \bar{x}^2$ and $\tilde{x} = \bar{x}/\sqrt{2}$.

The equilibrium conditions for the urban area with the head tax are

$$\frac{y - t\bar{x} - cz/n}{\ell} = r_A \tag{14}$$

$$n\ell = \pi\bar{x}^2 \tag{15}$$

$$\frac{u_2(\ell,z)}{u_1(\ell,z)} = \frac{c\ell}{n(y - t\bar{x}/\sqrt{2}) - cz} \tag{16}$$

$$u = u(\ell,z) \tag{17}$$

Equation (14) says that land rent at the urban periphery equals agricultural rent, and the other conditions are already familiar. Since there are five unknowns, ℓ, z, u, n, and $\bar{x}$, and four equations, either u or n must be exogenous. In the head tax equilibrium, we have seen the emergence in a spatial context of the median voter principle, which is familiar from non-spatial models of public goods provision (see Barr and Davis).

II.

In this section, we compare the head tax and property tax equilibrium solutions. Since the comparison is impossible for a general utility function, computations with specific functions were attempted. When $r_A > 0$, comparison of the property and head tax solutions using a Cobb-Douglas utility function is impossible. Solution of the systems yields expressions which do not allow unambiguous comparisons of the equilibrium values of the variables. However, when $r_A = 0$, the equation systems are simplified and complete comparisons of the head and property tax equilibria in both the open and closed city cases are achievable with a Cobb-Douglas utility function. The CES function allows a comparison of the equilibria in the closed city case, although no results are available for the open city case. Since the CES results are incomplete, only the Cobb-Douglas solutions are presented below.

When $r_A = 0$ and the utility function is $u = \ell^{\alpha} z^{\beta}$, the property tax equilibrium system (7) - (11) becomes

$$y - t\bar{x} = 0 \tag{18}$$

$$n\ell = \pi\bar{x}^2 \tag{19}$$

$$\frac{\beta\ell}{\alpha z} = \frac{c\ell}{V + cz} \tag{20}$$

$$\pi y\bar{x}^2 - 2\pi t\bar{x}^3/3 = \ell(V + cz) \tag{21}$$

$$u = \ell^\alpha z^\beta, \tag{22}$$

where, in (21), the integration in (9) has been performed. The head tax system (14) - (17) becomes

$$y - t\bar{x} - cz/n = 0 \tag{23}$$

$$n\ell = \pi\bar{x}^2 \tag{24}$$

$$\frac{\beta\ell}{\alpha z} = \frac{c\ell}{n(y - t\bar{x}/\sqrt{2}) - cz} \tag{25}$$

$$u = \ell^\alpha z^\beta. \tag{26}$$

For the open city case, u is exogenous and was set equal to unity. Since solving (20) for z in terms of V yields $z = \beta V/(\alpha-\beta)c$, a positive value for z requires $\alpha > \beta$. Solving for ℓ in terms of z from (22) with $u = 1$ yields $z = \ell^{-\alpha/\beta}$ which allows V to be expressed in terms of ℓ. Then, using (18), (21) can be written in terms of ℓ and parameters, yielding

$$\ell_P = \left(\frac{\pi y^3}{ct^2}\right)^{\frac{\beta}{\beta-\alpha}} \left(\frac{\beta}{3\alpha}\right)^{\frac{\beta}{\beta-\alpha}}, \tag{27}$$

where the subscript P refers to the property tax equilibrium. Solution of (23) - (26) using similar substitutions yields

$$\ell_H = \left(\frac{\pi y^3}{ct^2}\right)^{\frac{\beta}{\beta-\alpha}} \left\{ \left(\frac{\alpha}{\alpha+\beta(1-1/\sqrt{2})}\right)^2 \left(1 - \frac{\alpha}{\alpha+\beta(1-1/\sqrt{2})}\right) \right\}^{\frac{\beta}{\beta-\alpha}}, \tag{28}$$

where H denotes the head tax equilibrium. Tedious calculation shows that $\beta/3\alpha$ exceeds the expression in brackets in (28). Since these expressions are raised to a negative power, $\ell_H > \ell_P$; land consumption in the head tax equilibrium exceeds land consumption in the property tax equilibrium. Since the consumption bundle is on the unit indifference curve in both cities, $\ell_H > \ell_P$ implies $z_H < z_P$. The output of the public good is greater with the property tax than with the head tax. The solution of the head tax equilibrium also yields

$$\bar{x}_H = \frac{y}{t}\,\frac{\alpha}{\alpha+\beta(1-1/\sqrt{2})} < \frac{y}{t} = \bar{x}_P\ , \tag{29}$$

where $\bar{x}_P$ is from (18). Since $\bar{x}_H < \bar{x}_P$ and $\ell_H > \ell_P$, $n_H < n_P$ from (19) or (24). Thus, when the city is open, the property tax results in lower land consumption, higher public good consumption, larger urban area, and larger population than does the head tax.[4]

When the city is closed, u is free to vary but n is fixed. Equation (29) still holds, which since n is fixed, implies $\ell_H < \ell_P$ using (19) or (24). Solving for z_H and z_P in terms of $\bar{x}_H$ and $\bar{x}_P$ yields $z_H = n(y-t\bar{x}_H)/c$ and $z_P = \beta n(y-2\bar{x}_P/3t)/\alpha c$. After substituting for $\bar{x}_H$ and $\bar{x}_P$ from (29), $z_H < z_P$ requires

$$\frac{\beta(1-1/\sqrt{2})}{\alpha+\beta(1-1/\sqrt{2})} < \frac{\beta}{3\alpha}, \tag{30}$$

an inequality which can easily be established. Thus, when the city is closed, the property tax leads to higher land consumption, higher public good consumption and larger urban area than does the head tax. Since consumption of both goods is greater with the property tax, $u_P > u_H$; the closed city reaches a higher utility level with the property tax than with the head tax. Presumably, the inefficiency of the head tax is related to the lack of unanimity among voters in the head tax equilibrium. The results of this section are summarized in the top half of Table 1.

III.

The next step in the analysis is the characterization of the optimal city under the different tax regimes, and a comparison of the optimum to the majority voting equilibrium. Clearly, optimization makes sense only in the closed city case. The optimality conditions under the property tax regime with $r_A > 0$ are extremely complex, preventing comparison of the majority voting equilibrium and the optimum. When $r_A = 0$, the optimality conditions lead to a nonsensical solution with $V = 0$. For these reasons, the analysis in this section is concerned only with the optimal city under the head tax regime. In computing the optimum, the welfare of the absentee landlords is not considered; the goal is to maximize the utility level of the urban residents only. The analysis is carried out for arbitrary agricultural rent r_A.

The problem is to maximize $u(\ell,z)$ subject to the constraints $n\ell = \pi\bar{x}^2$ and $y - t\bar{x} - cz/n = r_A\ell$. The first-order condition is

$$n\frac{u_2(\ell,z)}{u_1(\ell,z)} = \frac{c}{(r_A + \frac{t\bar{x}}{2\ell})} \tag{31}$$

The interpretation of (31) is straightforward. Land rent $r(x)$ equals $(y - tx - cz/n)/\ell$, which can be expressed $r_A + t(\bar{x} - x)/\ell$ using the second constraint. Expenditure on land at x, $r(x)\ell$, equals $r_A\ell + t\sqrt{n\ell/\pi} - tx$, using the first constraint. Differentiating the last expression with respect to ℓ yields the denominator of the RHS of (31). Thus, (31) says that the ratio of the marginal cost of z to the marginal cost of ℓ equals the sum of the marginal rates of subsituation between z and ℓ, the familiar Samuelson condition.

The optimal city is characterized by (31) and the two constraints. Since (31) bears no relation to (16) in the head tax equilibrium system, the majority voting equilibrium does not yield the maximal value of u. It is interesting to note, however, that if consumers recognize the dependence of land rent on their consumption decision, so that the budget constraint is perceived as $y - r_A\ell - t(\sqrt{n\ell/\pi}-x) - cz/n = 0$, then consumer maximization results in the first-order condition (31), and consumers are unanimous over the optimal consumption bundle since (31) does not depend on x. This result is of course not surprising; it says that if consumers engage in non-competitive behavior, they can raise their level of utility. Wile has noticed a similar fact in his analysis, but he misleadingly attributes the result to the consumers' failure to account for the spatial externality they impose on others: higher land consumption by one individual imposes higher transportation costs on others due to the expansion of the city. The correct explanation is that consumers can reach a higher utility level when they recognize the influence of their consumption on the prices they pay than when they ignore this effect.

It is possible to compare the majority voting equilibrium and the optimum using a CES utility function.[5] For the CES function $u = (\alpha\ell^{-\rho} + (1-\alpha)z^{-\rho})^{-1/\rho}$, the marginal rate of substitution u_2/u_1 is $((1-\alpha)/\alpha)(\ell/z)^{\rho+1}$. Solving the head tax equilibrium system for ℓ yields

$$\frac{1-\alpha}{\alpha} \ell^{\rho+1} (r_A + t(1 - 1/\sqrt{2})\sqrt{n/\pi\ell}) = (\frac{n}{c})^{\rho} (y - t\sqrt{n\ell/\pi} - r_A\ell)^{\rho+1} \quad (32)$$

Solving for the optimum we have,

$$\frac{1-\alpha}{\alpha} \ell^{\rho+1} (r_A + t\sqrt{n/\pi\ell}/2) = (\frac{n}{c})^{\rho} (y - t\sqrt{n\ell/\pi} - r_A\ell)^{\rho+1} \quad (33)$$

It can be deduced as follows from (32) and (33) that $\ell_{OP} < \ell_{MV}$ when $\rho > -\frac{1}{2}$, where OP and MV denote optimum and majority voting. It is easy to show that for $\rho > -\frac{1}{2}$, the LHS of (32) and the LHS of (33) are increasing in ℓ. The RHS of these equations is decreasing in ℓ, since $\rho > -1$, and for any ℓ the LHS of (33) exceeds the LHS of (32). This means that, on the downward-sloping curve corresponding to the common RHS of (32) and (33), the intersection of the curve representing the LHS of (33) is uphill from intersection of the curve representing the LHS of (32). This establishes $\ell_{OP} < \ell_{MV}$. Since n is fixed, $\ell_{OP} < \ell_{MV}$ implies $\bar{x}_{OP} < \bar{x}_{MV}$. In addition, since we know $u_{OP} > u_{MV}$, it must be true that $z_{OP} > z_{MV}$ given that $\ell_{OP} < \ell_{MV}$. These results clearly hold for the Cobb-Douglas utility function $\ell^{\alpha}z^{1-\alpha}$, which corresponds to the case $\rho = 0$, and it is easy to show they also hold for $u = \ell^{\alpha}z^{\beta}$, $\beta \neq 1 - \alpha$. Since the elasticity of substitution σ equals $1/(1+\rho)$, the condition $\rho > -\frac{1}{2}$ is equivalent to $\sigma < 2$. The conclusion is that as long as $\sigma < 2$, or ℓ and z are not too substitutable in consumption, the optimal city has higher public good consumption, lower land consumption, and smaller urban area than the city which emerges in the majority voting equilibrium. These results are summarized in the bottom half of the Table 1. Presumably, land consumption in the optimal city is lower because

the effect of the consumption level on land rent is taken into account.

The failure of the head tax equilibrium to maximize urban utility does not mean the equilibrium is Pareto inefficient. This is true because aggregate rent payments are less at the optimum than in the head tax equilibrium; urban residents are better off at the optimum but absentee landlords suffer a loss of income. To see this, note that expressing $r(x)$ as $r_A + t(\bar{x} - x)/\ell$ and computing aggregate rent, $\int_0^{\bar{x}} 2\pi x r(x)dx$, yields, after substituting for ℓ , $\pi r_A \bar{x}^2 + nt\bar{x}/3$. Since aggregate rent is increasing in $\bar{x}$ and $\bar{x}_{OP} < \bar{x}_{MV}$, the income of absentee landlords is less at the optimum than in the head tax equilibrium.

IV.

In this section, we consider a closed city where $r_A = 0$ and the income of each consumer is augmented by an amount V/n, his share of aggregate land rent. The most natural interpretation of this assumption is that the government owns all the land in the area and redistributes aggregate rents equally to consumers. A less satisfying interpretation is that each of the n consumers owns a pie slice of land of angular width $2\pi/n$ that extends indefinitely outward from the CBD.

The budget constraints of consumers under the property tax and head tax respectively are $r\ell(1 + cz/V) = y - tx + V/n$ and $r\ell + cz/n = y - tx + V/n$. As before, consumers are perfectly competitive with respect to V. The first-order conditions for utility maximization under the property tax and the head tax are (10) and $u_2/u_1 = c/(n(y - tx) - cz + V)$ respectively. As before, the property tax equilibrium is characterized by voter unanimity, while in the head tax equilibrium, the first-order condition holds only at $x = \tilde{x}$. Other familiar

equilibrium conditions require that land rent equal zero at $\bar{x}$, $n\ell = \pi\bar{x}^2$, and $\int_0^{\bar{x}} 2\pi x r(x)dx = V$. Substituting for $r(x)$ from the budget constraints, it is easy to show that the last condition is equivalent to $cz/n = y - 2t\bar{x}/3$ under both the property tax and head tax regimes. Thus, the property tax equilibrium system is

$$y - t\bar{x} + V/n = 0 \tag{36}$$

$$n\ell = \pi\bar{x}^2 \tag{37}$$

$$cz/n = y - 2t\bar{x}/3 \tag{38}$$

$$\frac{u_2(\ell,z)}{u_1(\ell,z)} = \frac{c\ell}{V + cz} \tag{39}$$

The head tax equilibrium is characterized by (37), (38), $y - t\bar{x} + (V - cz)/n = 0$, and $u_2/u_1 = c/(n(y - t\bar{x}/\sqrt{2}) - cz + V)$. Equations (37) and (38) define a locus of points $cz/n = y - 2t\sqrt{n\ell/\pi}/3$ which contains the consumption bundles for both tax regimes. The optimal bundle is the point where the locus is tangent to an indifference curve,[6] or where

$$\frac{u_2(\ell,z)}{u_1(\ell,z)} = \frac{3c\ell}{nt\bar{x}} \quad . \tag{40}$$

Clearly the optimal (ℓ, z) bundle is the same for both tax regimes. What differs between the regimes is the value of V, which is derived from the appropriate boundary condition on r once the optimal bundle is known.

It follows directly from (36) - (39) that the property tax equilibrium is identical to the optimum, a striking result. The RHS of (39)

becomes $3c\ell/nt\bar{x}$ after substituting for V and z from (36) and (38). It is easy to see by examining the head tax equations that the head tax regime does not generate the optimum.

The efficiency of the property tax in the model with rent redistribution invites an analogy between the property tax equilibrium and a Lindahl equilibrium from the theory of public goods. In the Lindahl equilibrium, consumers with different tastes and incomes are charged different unit prices for the public good which are designed to lead each consumer to demand the same socially optimal level of the good. The form of the budget constraint with the property tax leads to the same kind of unanimity among consumers in the urban area by eliminating the effect of location on the desired consumption bundle, and the structure of the model guarantees the equivalence of the optimal and desired bundles. It is easy to show, however, that this equivalence disappears when $r_A > 0$. Apparently, the property tax becomes inefficient when the model is not "closed," when productive activity generating positive land rent occurs outside the city.

V.

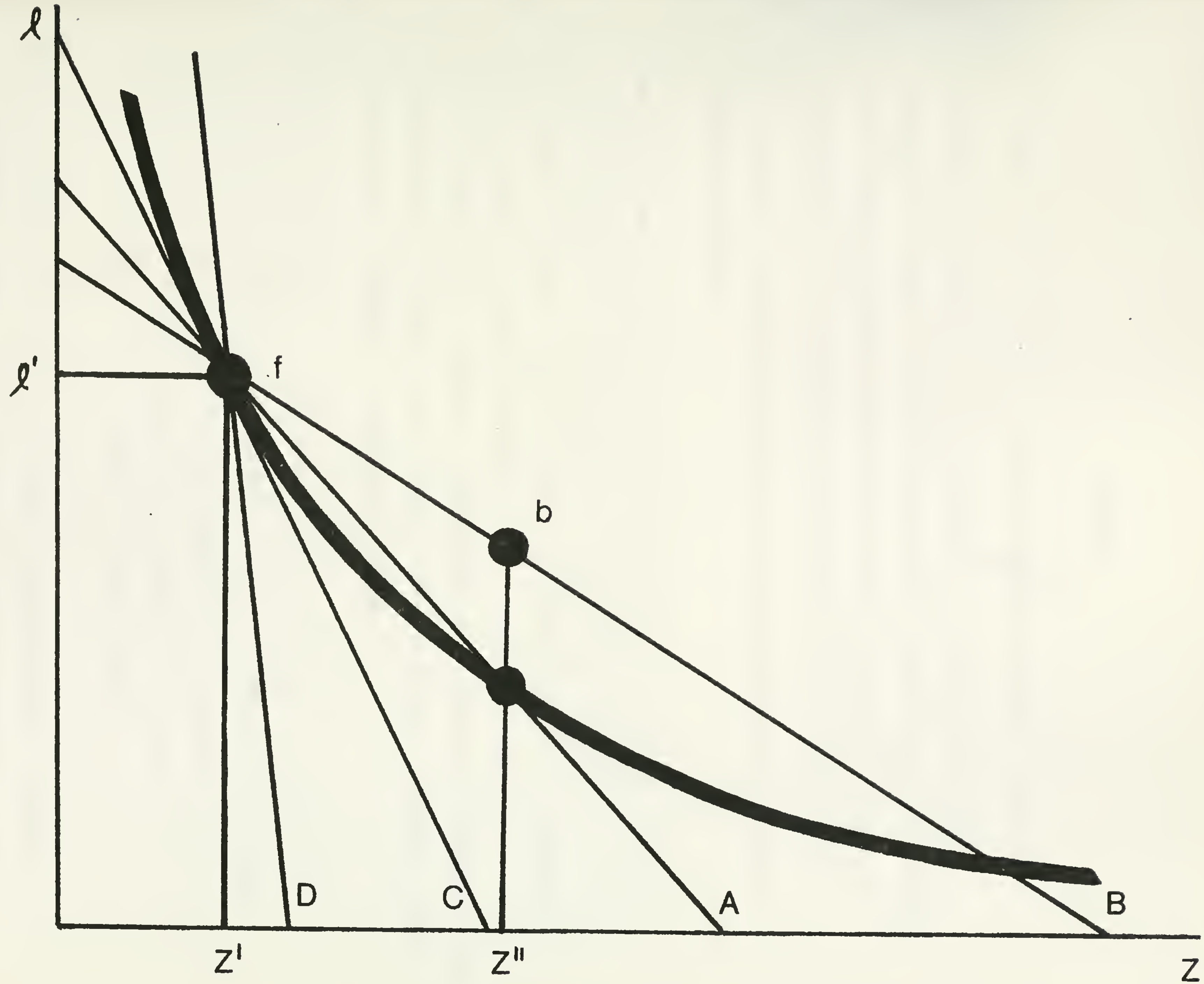
This paper shows that the analysis of majority voting equilibria in an urban spatial context can generate a number of interesting results, further evidence of the rich theoretical implications of the simple urban spatial model. In addition, the analysis has demonstrated the ease with which the notion of a majority voting equilibrium may be applied to yield results of practical significance.

The models presented in this paper are unrealistic because the public good is not subject to congestion and because only two goods are consumed.

Nevertheless, if we are willing to grant that the models capture some essential features of reality, there is a public policy lesson inherent in the results: in a world where migration is difficult and cities may be viewed as closed, a property tax system may be inherently superior to a head tax regime (or to an income tax, which is equivalent to a head tax when incomes are equal) as a mechanism for raising revenue for public expenditure. Unanimity among voters and a higher urban utility level result from the property tax system, regardless of land ownership arrangements. Happily, municipalities rely heavily on the property tax as a source of revenue for public expenditures. Hence, present institutional arrangements cannot be faulted on efficiency grounds, at least from the perspective of the above analysis.

Table 1.

COMPARISON		ℓ	z	$\bar{x}$	n	u
Property Tax vs. Head Tax. Cobb-Douglas Utility Function	open city	$\ell_P < \ell_H$	$z_P > z_H$	$\bar{x}_P > \bar{x}_H$	$n_P > n_H$	$u_P = u_H$
	closed city	$\ell_P > \ell_H$	$z_P > z_H$	$\bar{x}_P > \bar{x}_H$	$n_P = n_H$	$u_P > u_H$
Optimum vs. Majority Voting Equilibrium, with Head Tax	CES utility function, $\rho > -\frac{1}{2}$	$\ell_{OP} < \ell_{MV}$	$z_{OP} > z_{MV}$	$\bar{x}_{OP} < \bar{x}_{MV}$	$n_{OP} = n_{MV}$	$u_{OP} > u_{MV}$


ℓ
ℓ'
f
b
D
C
A
B
Z'
Z''
Z

FOOTNOTES

[1]The author wishes to thank an anonymous referee for helpful suggestions.

[2]The second derivative of (1) is $2c^2\ell/(V+cz)^2 > 0$, which means the budget constraint is convex. The second-order condition requires that the curvature of the indifference curve be greater than that of the budget constraint at the solution to (3). We assume the second-order condition is satisfied.

[3]In this case the second-order condition requires only the usual convexity of indifference curves.

[4]Fortunately, it is possible to show that the second-order condition for the consumer equilibrium with the property tax is satisfied when $\alpha > \beta$. The rate of change of the slope of the Cobb-Douglas indifference curve is $\beta\ell(\alpha+\beta)/(\alpha z)^2$. This must exceed the second derivative of the budget constraint at the solution for the solution to be a maximum. From footnote 1 this requires $(\alpha+\beta)/\alpha z > 2c/(V+cz)$, where (20) has been used to cancel terms. Substituting for z in terms of V from the property tax solution, the inequality reduces to $\alpha > \beta$.

[5]Since the constraint $y - t\sqrt{n\ell/\pi} - r_A\ell = cz/n$ is convex, the second-order condition requires that the curvature of the indifference curve exceed the curvature of the constraint at the solution to (31). However, it is not possible to verify whether the second-order condition is satisfied for the CES solution. All we can do is assume the condition holds.

[6]Since the constraint $cz/n = y - 2t\sqrt{n\ell/\pi}/3$ is again convex, the second-order condition requires that the now familiar curvature condition holds at the optimum.

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